

## Physics and complexity

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Lest I would be accused of lese-complexity in the following, let me open my discussion with a warning by one of the most prominent promoters of the idea:

« The word “complexity” cannot but express our embarrassment, our confusion, our inability to define in a simple manner, to name with clear terms, to put order in our ideas (...). Complexity is a problem-word, not a solution-word. »

Edgar Morin, *Introduction à la pensée complexe* (1990)

Whether the author has consistently followed his own admonishment remains an open question...

### Is there a physics of complexity?

Starting a few decades ago, the theme of complexity became very fashionable in many writings about contemporary science and has not ceased to be since, giving rise to various and dubious extrapolations. Let me concentrate here on the place and role, if any, of complexity in physics. The following quotation is representative of many assertions about this thematic:

«...it is at the heart of physics - which dreamed of a well-fixed, well-ordered world - that the complex came to bring its disorders. From thermodynamics to the general structure of the cosmos, via microphysics, integration and disintegration were found everywhere.» *Le Monde aujourd'hui*, 3-4 June 1984, comment about a symposium on the theme of complexity.

One cannot help being struck by the daring assimilation of complexity with disorder, or the curious use of the overly versatile terms “integration” and “disintegration”. It seems that physics, as a proclaimed exact science, is here summoned as a witness of epistemological morality in support of a notion whose potential importance and effectiveness - as the rest of the article clearly showed - is supposed to concern

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mainly the biological and social sciences. The physicist, however, cannot recognize in these few lines a relevant description of the present evolution of his discipline. Complexity is not really a keyword in current physics and does not correspond to a specific concept, an explicit problem or even a research program<sup>1</sup>. It is telling, as an empirical proof, to browse through the list of publications of the Santa Fe Institute, which is the self proclaimed Mecca of complexity; for the year 2019, out of roughly 120 papers produced at the Institute, less than 20 dealt with physics; furthermore, most of these played only lip service to complexity and were rather standard works in statistical physics or condensed matter physics<sup>2</sup>.

It must be recognized of course that some reputable authors have tried to give various definitions of complexity which might be relevant for physics, C.B. Bennett being one of the foremost proponents<sup>3</sup>, followed, among others, by M. Gell-Mann<sup>4</sup>. However, such attempts almost immediately were received with a strong, if not unanimous, scepticism<sup>5</sup>. The critics pointed out the very narrow field of application of such definitions. It is true, indeed, for example, that algorithmic complexity (the so-called Solomonoff-Kolmogorov complexity), a well-defined mathematical notion, is used in information and computation theory. As far as physics is concerned, algorithmic complexity has found some applications in the field of statistical physics, for instance in discussions about the vexed notion of entropy<sup>6</sup>. However, precisely because of its rigorous and specific mathematical definition, which concerns but sequences of integer numbers, its relevance is strictly limited to that domain. One might anyway ask whether the terminological choice of “complexity” to name the notion is really appropriate (note that neither Solomonoff nor Kolmogorov used the term “complexity”<sup>7</sup>); indeed, algorithmic complexity is a purely quantitative measure, far away from the qualitative meanings currently associated with the epithet “complex” as usually attributed to some concrete phenomena or theoretical analyses, appealing to ideas such as emergence, multiplicity, interconnection, feedback, recursivity, etc. The inappropriateness of the term is best exemplified by the fact that the algorithmic complexity of a sequence of numbers is maximal when this sequence is absolutely random, that is, without any inner structure. Similar remarks could be made about other various definitions of complexity measures. A recent and heavily documented critical review has been offered by Cosma Rohilli Shalizi who goes so far as to state that

« every few months seems to produce another paper proposing yet another

measure of complexity, generally a quantity which can't be computed for anything you'd actually care to know about, if at all. These quantities are almost never related to any other variable, so they form no part of any theory telling us when or how things get complex, and are usually just quantification for quantification's own sweet sake. »<sup>8</sup>

Let me only add, in order to keep the record straight, that some people like to think that a fundamental theory of physics would ultimately be based on simple abstract relations between some discrete elements<sup>9</sup>. The simplest example would be that of a Universe consisting of (or described by) a cellular automaton, such as a sophisticated version of Conway's "Game of Life". This would lead to an essentially computational world view, where algorithmic complexity of one type or another might come to play a crucial role in what would be a radically new type of physics<sup>10</sup>. But the irony of the situation is that such a program, whatever its merits, is aiming at finding a most simple algorithm, with a complexity measure as low as possible!

A notable example of the necessity to keep some distance from an uncritical acceptance of complexity as an all-encompassing notion is furnished by the introduction of a 1994 Colloquium on complexity in physics<sup>11</sup>. The very organiser of the Colloquium, the outstanding physicist Philip W. Anderson (Nobel prize 1977), while using in its introduction "complexity" as a catchword, explicitly and impishly develops the idea that, in fact, it is another idea that has to be brought forward and studied, namely "emergence". Another strong and convincing warning against of the hopes aroused by the notion of complexity has been offered by the mathematician and philosopher of science Giorgio Israel, who summed up his misgivings by stating that

« the science of complexity proposes forms of reductionism that are even more restrictive than the classical ones, particularly when it claims to unify in a single treatment problems that vary widely in nature such as physical, biological, and social problems »<sup>12</sup>.

As for now, the term "complexity" as related to physics is used more or less wisely in epistemological exegesis or journalistic commentary to evoke the renewal and diversification of certain investigative themes, such as, for example, the return in force of studies on macroscopic phenomena, from whirlpools in bathtubs to sand heaps. But perhaps it would suffice to say that physics today recognises (empirical) complication without tackling (theoretical) complexity, which makes it difficult for

the physicist to come and lay his obole at the new idol. He is all the more deterred to pay tribute to it in his field by his rather systematic mistrust of the numerous overly general notions, with interdisciplinary vocation and global pretensions, which are regularly proposed as panaceas to compensate for the diversification of knowledge. The forced unification of knowledge is probably even more sterile than its spontaneous dispersion. The diversity of disciplines is a guarantee both of their interest and of their proper use, and the frequent calls for interdisciplinarity and a recomposition of sciences too often amounts to a derisory and sometimes dangerous fantasy<sup>13</sup>.

### **The decomplexity of theoretical physics**

How then to justify the relative indifference of physics to the idea of complexity? The fact is that a physically useful notion of complexity certainly would require going beyond the notion of sheer algorithmic or computational complexity, which, as we have pointed out, only deals with strings of numbers. Indeed, the physicist would like to apply the notion to his objects of study themselves rather than to the numerical formalism used in his theories. In other terms, does there exist a definition of complexity which would allow us to decide whether an atom is more or less complex than a clockwork, or a quasar than a biomolecule? To try answering this question, let us adopt a tentative definition of complexity which seems just precise enough not to sink immediately into triviality, and just general enough to cover the cases where the relevance of the notion might be effective:

A system will be said to be “complex” if it exhibits  
reciprocal couplings between different levels or parts.

In this conception, in order to be able to speak of complexity, it is necessary 1) that a structure showing a plurality of hierarchical levels or a multiplicity of disanalogous components be distinguished, but 2) that the inner connexions of this structure be not linear, one-way, but mutual. In the words of Jean-Pierre Dupuy :

« A key term, synonymous with complexity, is that of autotranscendance: the upper level “buckles back” on the lower level from which it arises. »<sup>14</sup>

In other words, it is the conjugation of both *structural heterogeneity* and *functional reciprocity* that seems to underlie the notion, and could lead to give it a defined content. This definition has the advantage of applying to any type of system, whether

material (a natural object) or intellectual (a theoretical construction), whatever the nature, size or number of its elements.

But, therefore, almost any real system is obviously complex, whether it is an oil refinery or a hydrocarbon molecule, the theory of heredity or a living cell. And we are in danger, if we accept this definition, of falling back into soft universality. The point in fact is that we are able to escape triviality, because some systems can be described and understood precisely by ignoring or neglecting their inherent complexity. It is therefore less the relevance of complexity that is of interest here, than the limits of its usefulness. For it is indeed one of the major achievements of physics to succeed in treating so many small and large parts of the Universe as if they were *not* complex, by subjecting them to linear and/or homogeneous patterns that escape the circularity and/or heterogeneity that constitute complexity - according to the definition we have proposed.

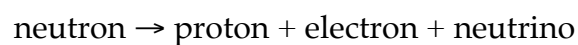
Let us consider the fundamental, and indeed founding, example of classical mechanics. The trajectory of an object - let us assume it to be pointlike for the sake of simplicity - in a given field of forces is governed by Newtonian dynamics according to the following scheme: the value of the field at the object's position for a given instant determines the force acting on it, which, according to Newton's law, gives the acceleration, that is, the change in its velocity, and therefore the subsequent position, etc. There is indeed, in this statement, a mutual coupling between the two heterogeneous notions of force and position, a characteristic of complexity as we have defined it. Now, physics may claim to be the exact science *par excellence* only by virtue of its mathematisation<sup>15</sup>; it thus transforms the previous verbal statement into a formula:

$$m \frac{d^2 r}{dt^2} = F(r, t).$$

It is not by pedantry that this differential equation is written here, but because it exemplifies the very operation by which theoretical physics dissipates the initial complexity of the problem. The mathematisation of the notions of force and acceleration purifies them from their empirical concreteness and transcribes them into abstract quantities which may be related through some numerical constant expressing a conceptual equivalence — that is, the mass in the present case. Force and acceleration thus find a homogeneity that is clearly demonstrated by the

equation that links them. Likewise, the apparent paradox of the question (to know the trajectory, we need to know the acceleration, therefore the force, which depends on the trajectory, etc.) is solved by the general theory of differential equations, which assures us of the existence of a well-defined solution, once the initial conditions (position and speed) are known - and even provides us with the means to calculate it effectively.

Here is another concrete, and more modern, example borrowed from nuclear physics. Atomic nuclei are, as we have known for nearly a century, assemblages of (more) elementary "bricks", the nucleons, of which there are two species, protons and neutrons; thus, an oxygen nucleus is composed of 8 protons and 8 neutrons, a uranium nucleus of 92 protons and 146 neutrons, and so on. Some of these compounds are stable and persist indefinitely in their being, if left in peace. Others, more numerous, are unstable and mutate spontaneously: this is the phenomenon of radioactivity. In particular, beta radioactivity consists in the emission of an electron (and a neutrino) by a nucleus which, through the process, changes its nature. The simplest case is that of the neutron itself which, in isolation, decays spontaneously:



(in about a quarter of an hour). However, these very same neutrons, individually unstable, enter into the composition of stable nuclei. Thus the collective level (that of the nucleus) obviously depends on the individual level (that of the constituents, protons and neutrons) but in turn influences its properties — the instability of individual neutrons being inhibited by the collective nuclear structure they generate. «Isn't this a superb example of complexity?» will you then ask the physicist. «If you like...» does he reply evasively, finding little interest in such a convoluted verbal description. For he holds a concept which, by formalising the situation, allows it to be homogenised and to break a circularity the vice of which he cautiously avoids. Indeed, considering the energy of the system is enough to unravel the loop. The mass of the neutron is greater than those of the proton, the electron and the neutrino taken together, which gives it, according to the Einsteinian equivalence ( $E = mc^2$ ), a surplus of energy compared to these three particles; the disintegration of the neutron then allows it to dissipate this extra energy, thus explaining its instability. In the general case, it is the total mass of the nucleus, obtained by subtracting from the sum of the individual mass energies of the protons and neutrons the total mass of the nucleus, including the binding energy of these constituents, which governs the stability or

instability, depending on whether the nucleus has a total mass lower or higher than the masses of the possible products of its radioactive decay. It is therefore the homogenisation on the same theoretical level, via the concept of energy, of properties relating to different empirical levels - individual (the masses of nucleons) and collective (their binding energies) - that short-circuits the idea of complexity.

We could multiply the examples. They all show that where a description of the physical situation naturally evokes complexity (in the sense defined above), its analysis in fact avoids it. One could probably go so far as to claim that physical theory is based on the determined effort to eschew dealing with complexity as such. This objective may be thought of as a kind of paradigm, or at least a specific program of physics. The possibility of such an escape is intimately linked to the constitutive mathematisation of physics. But perhaps, in the end, it is, or will be, the same in other fields, where theoretical deepening (whatever its modes, not necessarily mathematical) might establish general concepts overcoming the heterogeneity and circularity of “complex” situations. Wouldn't the notion of complexity ultimately be a mirage, all the more frequent in the deserts of thought? Whatever its usefulness in characterising a phenomenon (neurobiological, sociological, etc.), establishing its interest and stimulating its investigation, complexity would thus dissipate in any analysis elaborate enough to account for the phenomenon - the fluidity of the heuristic notion evaporating in the aridity of formal theory, like those watery areas glimpsed far away in the desert that vanish at the approach of the thirsty researcher. The metaphor here is not necessarily meant to be deprecating: mirages can be useful - at least they make travellers go on, and sometimes in the right direction.

Let me conclude with the following statement, all the more interesting since it comes from human sciences, i. e. linguistics<sup>16</sup>:

« The scientific spirit requires that the complexity which he is presented with may be analysed in such a way as allowing the extraction of a single feature in order to use it as a key for the whole. »

L. Hjelmslev, *Prolegomena to theory of language* (1943)

Except that I would speak of a small number of features, rather than of a single one, I am afraid not to be able to contradict this assertion, at least concerning physics. But this is not the end of the story as physics is not a purely theoretical endeavour.

### **The complexity of practical physics**

Indeed reality *is* complex, even if theoretical physics tries to ignore it, and can thus move forward thanks to its very blinkers. It is thus hardly surprising to see the spectre of complexity, driven out of the object of physics, coming back to haunt its practice. In various respects, indeed, *complex* modes of operation - in the strong sense defined above - can be revealed; it is therefore understandable that, without contradicting the flat transparency of the statements of physical science, they result from processes that are less simple and linear than is often believed. Consider, for example, the different fields of physics usually presented according to a hierarchy of linearly embedded levels which seem to justify the common reductionism: particle physics underlying nuclear physics, itself the basis of atomic and molecular physics which serves as the foundation for condensed matter physics, etc. It is true, of course, that the laws operating at each of these levels underpin the material constitution of the next; the existence of nuclei is made possible by the nature of the interactions between the particles (nucleons) that constitute them. Even if it is recognised that this reductionism remains a statement of principle rather than an actual development (except in very special cases, we do not know how to explain concretely the detailed properties of the nucleus from the forces between nucleons, nor those of the molecules from the nuclei and the electrons), it serves as a theoretical basis for the erection of a hierarchy of values between disciplines, the noblest being identified with the most fundamental: particle physics thus claims pre-eminence over molecular physics or solid state physics, which are sometimes considered to be vulgarly applied physics<sup>7</sup>. This presumption, if naive, is hardly innocent, and its institutional and economic effects are major. The choice of priorities in research policies and the proportion of the budget devoted to each discipline are strongly affected. Compare, for example, the funding of particle physics and its titanic accelerators with that of condensed matter physics, which is so rich in potential technical applications. It is therefore not useless to somewhat belittle the arrogance of fundamental physics, by pointing out that, if some of its theoretical outcomes may contribute to other fields, it is on the other hand totally dependent on these other fields at the experimental level. As an example, the equipment of particle physics (detectors, counters, computers) is largely based on the discoveries and achievements of solid state physics (transistors, superconductors, etc.). Better still, the autonomy of the various levels remains such that each one continues to be a place of conceptual elaboration, and that a fundamental discipline often benefits from importing ideas



that have appeared in fields that it claims to dominate (thus statistical physics has largely fertilized particle physics). It is this mutual interaction between levels that we have called complexity. Taking it into account, would it be only at a heuristic level, should eschew too rapid value judgements and bring more flexibility in the management of research organisations and funds. These considerations, internal to physics, naturally extend to the relations between the major scientific disciplines. They would, for example, lead to a reevaluation of the status of chemistry in relation to physics, and, above all, of the social sciences and humanities in relation to the biological and physical sciences. We certainly need the former, more and more, to develop and control the latter.

In addition to this functional and organisational complexity of physics, one has to acknowledge its formal and conceptual complexity. Theorising in physics is not univocal. A given class of phenomena often may be analysed through different theoretical formulations, no doubt equivalent as far as this class is concerned, but with more or less broad domains of extension and very foreign basic concepts. Thus classical mechanics, that allows to calculate the trajectories of material points subjected to given forces, can be formalised in at least three ways: in the Newtonian mode, by differential equations, where the ideas of force (remote action) and acceleration are first; in the Lagrangian mode, through a principle of least action considering the trajectories globally and selecting the real trajectory among all the putative ones; in the Hamiltonian mode, via a system of partial derivatives equations, putting the laws of conservation at the foreground. These theorisations are fundamentally different, and, outside the field of classical mechanics, show very different relevances: the Lagrangian vision extends quite naturally into classical relativistic (Einsteinian) mechanics, whereas it is the Hamiltonian point of view that proves to be the most fruitful for the transition to quantum mechanics. The formal equivalences between these conceptions too often mask the intertwined articulations between their widely separate fundamental notions. The existence of such relations between heterogeneous formalisms is a good illustration of complexity in physical theory, provided that the full depth and diversity of its development is taken into account. Of course, this complexity of the theoretical structure of physics responds to that of the real world, the understanding of which requires resorting to alternative or even opposite notions, such as global and local, movement and permanence, elementary and compound, finite and infinite, coexisting and interacting, etc.<sup>18</sup> It is of

utmost importance to recognise this polymorphism, to appreciate it, to reinforce it - in at least two respects: firstly, on an epistemological level, since, as we have said, various (formally) equivalent formulations of the same theory will prove more or less fecund when it comes to building a new and deeper theory; secondly, on a pedagogical level and, more broadly, on a cultural level, since access to the mastery of such a theory is often facilitated, and its understanding enriched, by the diversity of its presentations.

Moreover, the complexities of physics, as we have sketched them out, are not only an interesting element of an a posteriori analysis for historical or philosophical reflections. A better understanding and an explicit consideration of these aspects would likely shed light on certain problems in the work of physics itself. For example, the debate on the foundations and interpretations of quantum theory owes much of its persistence and confusion to the difficulty of recognising the complexity of the relations between classical theory and quantum theory: while it is normal that the latter was initially questioned on the basis of the former and in its terms, which were those of current practice, the interesting problem today is to base the former on the latter. This reversal of the interrogation radically transforms the questions and opens up new spaces for a proper theory: considering quantum "non-separability" as a philosophical problem has for too long obscured a physical problem, to wit, showing that classical objects are (roughly) separable, even if their fundamental quantum constituents are not. It would be interesting to study from a similar point of view the discussion of the "anthropic principle"; the distinction of levels (the observer/the Universe) and the asymmetry of their relations (where, here too, epistemological analysis and theoretical deduction do not coincide) would undoubtedly help to clear up the confusion.

It must therefore be granted that physics, if not a science of the complex, is indeed a complex science. But this is hardly a deep statement, as it only shows that if it is certainly worthwhile to recognise complexity as a feature of scientific activity, the word here may hardly go beyond its meaning in ordinary parlance and does not itself acquire a scientific or philosophical content.

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<sup>1</sup> A 2018 CNRS report by Rémy Mosseri, « Les thématiques de la complexité au département MPPU (Mathématiques, Physique, Planète, Univers) du CNRS », July 2008 (private communication) clearly shows a very cautious approach.

<sup>2</sup> See <https://www.santafe.edu/research/results/papers> (consulted February 16, 2021)

<sup>3</sup> Charles H. Bennett, « How to Define Complexity in Physics and Why », in W. H. Zurek ed., *Complexity, Entropy, and the Physics of Information*, Addison-Wesley, 1991 [CRC Press, 2018] ; also in Niels Henrik Gregersen, ed., *From Complexity To Life: On the Emergence of Life And Meaning*. Oxford University Press, New York, 2003.

<sup>4</sup> Murray Gell-Mann, « What is Complexity ? », *Complexity* 1, pp. 16-19 (1995) ; see <https://onlinelibrary.wiley.com/doi/10.1002/cplx.6130010105>

<sup>5</sup> Mark Perakh, « On Defining Complexity », <http://www.talkreason.org/articles/complexity.cfm>, 2004, archived on <https://arxiv.org/abs/nlin/0701048>. See also Mark Perakh, *Unintelligent Design*, Prometheus Books, 2004, chapter 1, section « Complexity According to Dembski », pp. 58-63.

<sup>6</sup> See for instance W. H. Zurek, « Algorithmic randomness and physical entropy », *Phys. Rev. A* 40, 4731- 4751 (1989)

<sup>7</sup> It seems that after the seminal work of Solomonoff (1960) and Kolmogorov (1963), the first to introduce the term « complexity » was Martin-Löf in 1966 [Jean-Paul Delahaye, private communication].

<sup>8</sup> Cosma R. Shalizi, « Complexity Measures », 2017, <http://bactra.org/notebooks/complexity-measures.html>

<sup>9</sup> Such is the idea promoted by Stanley Milgram : <https://www.wolframphysics.org/>, as well as works by A. Illichinsky, K. Zuse, E. Fredkin, J. P. Crutchfield, F. Berto & al.

<sup>10</sup> I thank Jean-Paul Delahaye for stimulating exchanges on this point.

<sup>11</sup> Philip Anderson, « Physics: The Opening to Complexity », *Proc. Nat. Acad. Sci. USA* 92 (July 1995), pp. 6653-6654.

<sup>12</sup> Giorgio Israel, « The Science of Complexity : Epistemological Problems and Perspectives », *Science in Context*, 18 (3), 479-509 (2005).

<sup>13</sup> See the superb defense and praise for the diversity of knowledge by Paul Feyerabend, *Farewell to Reason*, Verso, 1987.

<sup>14</sup> Jean-Pierre Dupuy, *La catastrophe ou la vie*, Seuil, 2021, pp. 69

<sup>15</sup> See Jean-Marc Lévy-Leblond, « Why Does Physics Need Mathematics », in E. Ullmann-Margalis ed., *The Scientific Enterprise*, Kluwer Academic Publisher 1992, pp. 145-161.

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<sup>16</sup> L. Hjelmslev, *Prolegomena to a Theory of Language* (1943)

<sup>17</sup> For a strong and devastating attack on physical reductionism, see the classical paper by Philip W. Anderson, « More Is Different », *Science* 177 (August 1972), pp. 393-396.

<sup>18</sup> See Jean-Marc Lévy-Leblond, *Aux Contraires (L'exercice de la pensée et la pratique de la science)*, Gallimard, 1986.