Abstract of the digital contents: Optimization-based piecewise smooth denoising toolbox

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The developed optimization-based piecewise smooth denoising toolbox performs nonlinear filtering of multivariate signals, designed: i) to be intrinsically multivariate ; ii) to favor piecewise smooth denoised time series ; iii) to favor that discontinuities occur jointly across components. Thus, each filtered signal consists of a collection of piecewise smooth segments (e.g. piecewise linear) connected by sparse discontinuities (e.g. changes in the first derivative), whose locations are shared by all components.

Non linear filtering is achieved by the nonsmooth convex minimization of a functional designed as the sum of a data fidelity term, that ensures a minimal distance between original and denoised signals, modeled as a L2-norm, and a penalization term that enforces both piecewise smoothness of each denoised signal and colocations between components of the discontinuities, based on the L1-norm of a derivative of chosen order of the denoised signals. As an exemple, piecewise linear (smoothness of first order) denoising is obtained by constraining the L1-norm of the Laplacian (second-order derivative) operator. The colocation of the discontinuities is achieved by combining these L1-norms via a L2-norm across components, hence the L-1/2 norm. These two terms are balanced by an hyperparameter that enforces sparsity in the set of discontinuities.

Because the L1-norm of the penalization terms implies non differentiability, minimization cannot be achieved by Gradient-Descent algorithms. Instead, minimization is

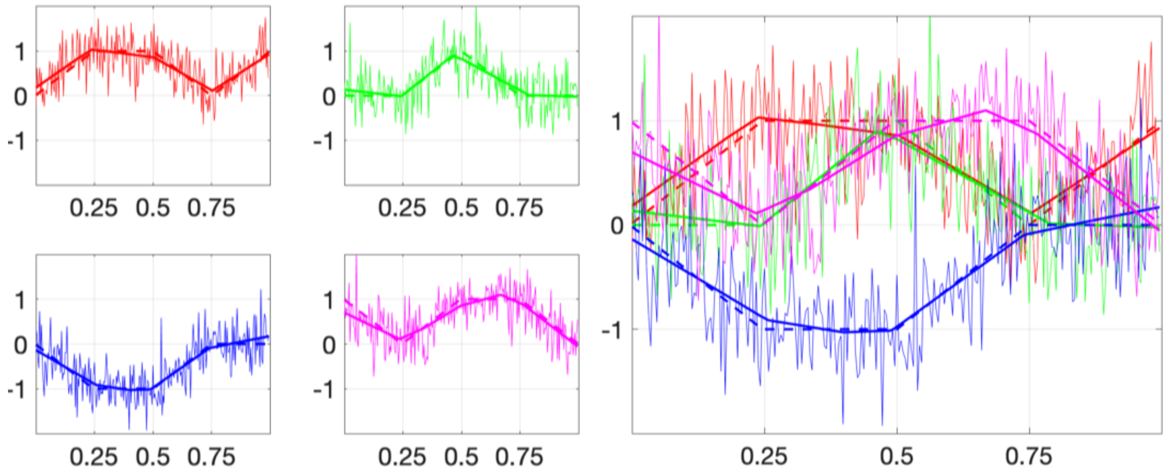

Figure 1: Example of 4-variate denoising (3 change points, 4 segments). Dotted lines: target; solid lines: result. Left: separate representations of the signal components, right: superimposed representations.

conducted using proximal operators (generalizing gradients to nondifferentiable functions). An accelerated iterative algorithm has been designed to ensure a fast, time and memory efficient and accurate optimisation.

The hyperparameter, controlling the sparsity of the set of discontinuities, is selected automatically by the data. This is achieved by a theoretical extension to multivariate dependent time series of the Stein Unbiased Risk Estimation (SURE) criterion, originally designed for univariate data. This automated data-driven hyperparameter selection is built-in the optimization, thus resulting in modest additional computational costs.

As a final feature of practical interest, the toolbox is designed to handle missing samples, which permits to filter signals of unequal lengths. As a nutshell summary, the procedure filters out noise fluctuations in multivariate data, except over a finite, sparse and data-selected subsets of locations where changes in the statistics of the data are actual and relevant (cf. Figure 1). The Matlab procedure, can be downloaded at https://github.com/bpascal-fr/stein-piecewise-filtering.